

Equations

A: Linear Equations (1 unknown)

1. $3t+5 = -t+1$ 2. $6.1 - (x+2.3)(3.2x-4.3) - 5.2x$
 $= 3.3x - x(3.1+3.2x) - 4.7$

3. $\frac{1}{2}\left(\frac{5}{8} + \frac{m}{4}\right) + 3 = \frac{1}{8}(3m - \frac{1}{3})$

4.

To solve $6x = 6$ cancel the 6's to get $x = 0$.

To solve $2x - 3 = 10$ divide both sides by 2 to get $x - 3 = 5$. Therefore $x = 8$.

$3x = 9$
I can transpose the 3 and get $x = 9 - 3$. So $x = 6$.

$\frac{x}{2} = \frac{2(x-1)}{3}$
I can eliminate the fractions by multiplying by the LCD which is 6. This gives $3x = 4(6x-6)$.

$\frac{x}{2} = \frac{x-5}{3}$
Multiplying by 6, the LCD, gives $3x = 2x - 5$

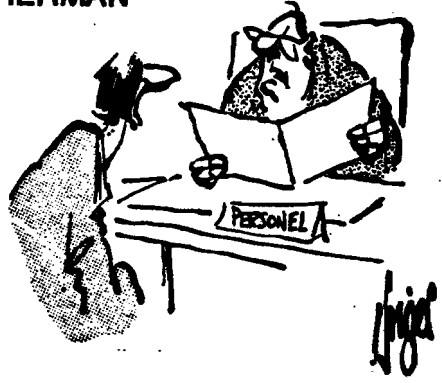


5. $(3 - 60d)(0.025d - 0.1) = 2.22 + 3.75d\left(\frac{2}{3} - 0.4d\right)$

6. $\frac{3x-4}{2} - \frac{3x-1}{16} = \frac{6x-5}{8}$

7. $(m-2)^2 - \frac{m-2}{5}$
 $= \frac{3m}{4} + \frac{m+2}{2} - (3 - m^2)$

HERMAN



Your resume says you spent "fore years at collej."



Can you spot 'spot' the dog?

BRAIN STRAINER:

Here are 4 marks:

||| |

Put down 5 more and make ten.

Prepared by Bill Habkirk

DAY

DAY

B: Equations with 2 and 3 unknowns

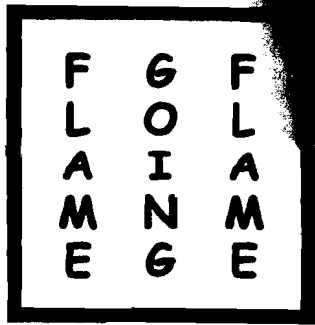
8. $5R - 6Q = 0$
 $85R + 42Q = 108$ [Use substitution for 8 & 9]

9. $3.25b + 107.5 = 29 - 5a$
 $0.015a - 10.08 = 0.2b + 43.8$

10. $\frac{m}{2} + \frac{n}{3} - 3 = 0$ 11. $4x - 2y = 3$
 $\frac{n}{2} + \frac{m}{5} = \frac{23}{10}$ $-2x + y = -2$

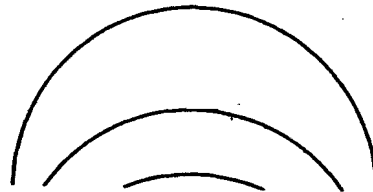
12. $5a + b - 4c = -5$ [Use addition
 $3a - 5b - 6c = -20$ or subtraction
 $a - 3b + 8c = -27$ for 10, 11 & 12]

13. $x + y = 35$
 $x + z = 40$
 $y + z = 45$



BRAIN STRAINER:
 $1 - (2 - (3 - (4 - \dots (9 - (10 - 11)) \dots)))$

C: Word Problems

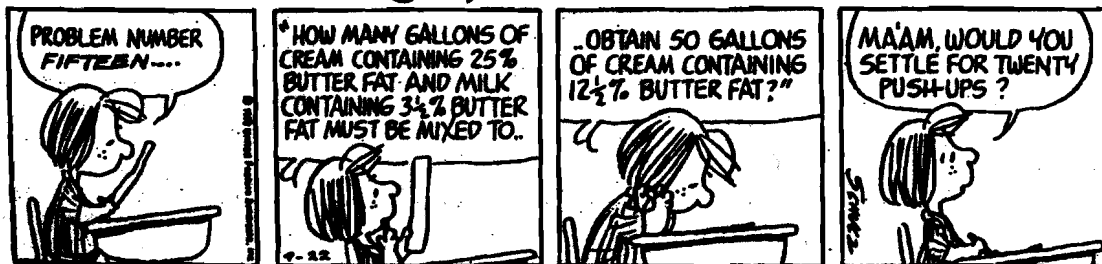


Which curve is sharpest?

14. Translate the following into equivalent algebraic equations in 1 unknown. Define the unknown clearly. No solution is necessary.

- (a) The mass of three-fourths of a bag of cement is equivalent to the mass of half a bag plus 10 kg.
- (b) Five-eighths of a number is equal to 10 times its square root.
- (c) Sixteen divided by some number equals that number less six.
- (d) A 60 m long rope is cut into 6 pieces, each being 1 m longer than the previous piece. A 3 m length remains.
- (e) Two numbers total 13 and differ by 7. (Give 2 solutions.)
- (f) The perimeter of a rectangle is 84 m. The length is 5 times its width.
- (g) The squares of 3 consecutive odd numbers total 8 more than 9 times the sum of the 3 numbers.

Solve 15 to 19 using equations in 1 unknown.



16. What are the angles in a triangle if the smallest is 2° more than $\frac{2}{3}$ of the middle angle and the largest is 12° less than 3 times the middle angle?

spectator sport.

Math is not a

BRAIN STRAINER

Monkey Business: A rope over the top of a pulley has the same length on each side. It weighs one third of a pound per foot. On one end hangs a monkey holding a banana, and on the other end a weight equal to the weight of the monkey. The banana weighs 2 ounces per inch. The rope is as long (in feet) as the age of the monkey (in years), and the weight of the monkey (in ounces) is the same as the age of the monkey's mother. The combined ages of the monkey and its mother are 30 years. One half of the weight of the monkey plus the weight of the banana is one fourth as much as the weight of the rope. The monkey's mother is half as old as the monkey will be when it is as old as its mother was when the monkey was when it was twice as old as its mother was when it was one third as old as its mother was when she was 3 times as old as it is now. How long is that banana?

17. A woman is presently 7 years older than twice her daughter's age. In 10 years the sum of their ages will be 75. Find their present ages.

18. A tank contains 4000 L of wine with an alcohol content of 10%. How much must be removed and replaced with wine with a 17% alcohol content to obtain wine with 12% alcohol?



19. A car starts out at 10:00 a.m. travelling at a constant speed. At noon a second car, travelling 24 km/h faster, starts from the same point to overtake the first car at 7:00 p.m. How fast must the 2nd car travel?

Solve 20 to 23 using equations in 2 or 3 unknowns.

20. \$10 000 is invested, part at 4% and the rest at 5% interest. The annual interest from the 4% investment is \$40 more than that from the 5% investment. Find the amounts invested at 4% and 5%.

21. There is a fraction such that if 3 is added to the numerator its value will be $\frac{1}{3}$, and if 1 is subtracted from the denominator its value will be $\frac{1}{5}$. Find the fraction.

22. One gasahol blend has 5% alcohol. Another has 11% alcohol. How much of each must be mixed to obtain 500 L of gasahol containing 9.5% alcohol?

I feel terrible. I can do all the algebra - use exponents, multiply terms, solve equations - but I still can't do word problems. What's wrong with me?

Nothing. You need to learn to do word problems; it doesn't come automatically when you learn algebra. It's a skill like riding a bicycle or playing a guitar.

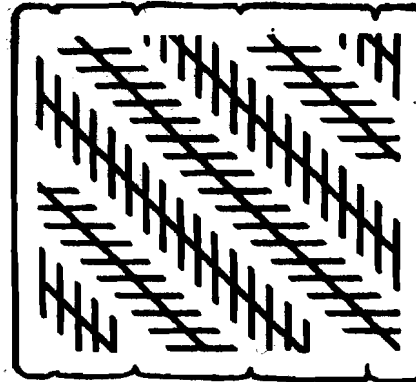
The equal sign was introduced in 1551 by Robert Recorde (English), a year before he died in prison.
"I will sette as I doe often in woorke use, a paire of paraleles, or twin lines of one lengthe, thus: =, bicause noe 2. thynges, can be moare equalle."

Problem Solving - what you do when you don't know what to do.

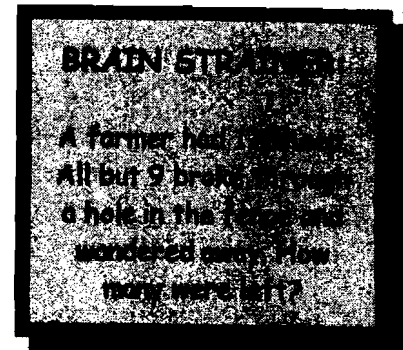
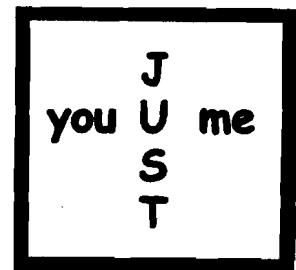
23. In a triangle ABC, angle A is 20° more than half the sum of angles B and C, and angle B is 5° less than twice angle C. Find the 3 angles.

Tips for Solving Word Problems

- read the question carefully (maybe 2 or 3 times)
- draw a diagram (if applicable) and show known data
- choose a letter to represent the unknown and make an introductory statement defining the unknown exactly (include units, if any)
Example: Find the area (in hectares) of a field whose length is five times its width and whose perimeter is 960 m.
 - in the example you might begin with the statement:
 Let the width of the field = W (in m)
- when working with only 1 unknown, as we are here, you should then define any other unknowns in terms of the defined quantity:
 ∴ the length of the field = 5W (m)
- if you can't define the unknown properly (as above), you probably can't complete the next step properly
 - see example 18 on pg. 98 in text for poor definitions of an unknown
- now write an equation using the unknown (easy to say, eh?)
 $2(W + 5W) = 960$
 and solve it: → $W = 80$
 - make sure the units balance in your equation
- don't forget to check the answer in the original equation (it checks)
- evaluate the unknowns:
 ∴ the width = 80 m and the length = 400 m
- check that the values satisfy the conditions of the given problem:
 - perimeter = $80 + 80 + 400 + 400 = 960$ m ✓
 - 5 times the width = $5 \times 80 = 400$ m = length ✓
- state the final answer to the problem
 - in this case an additional calculation is required
 - area = $80 \text{ m} \times 400 \text{ m} = 32\,000 \text{ m}^2 = 3.20 \text{ ha}$
 - a final statement might be: The area is 3.20 ha.
- in most word problems, there is no extraneous data (i.e. all given information must be used to obtain a solution)
- You must do many word problems if you expect to become proficient. But don't expect to solve every question in 4 or 5 minutes. It may take an hour with much confusion and many dead-ends along the way. Resist the temptation to ask someone too quickly how an 'unsolvable' problem can be solved; leave it for an hour or a day and try again. You may generate a lot of garbage before finding success.



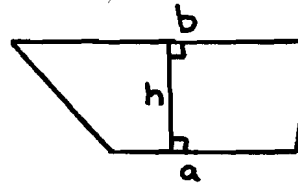
Are these long diagonal lines straight?



If you say good-bye to Saturn - and head out, what do you say hello to?

D: Formula Manipulation (Literal Equations)

24. The area of a trapezoid is given by the formula $A = \left(\frac{a+b}{2}\right)h$. Solve for h and then for a .



25. Solve for r : $\frac{I}{3} = \frac{2E}{R+2r}$

26. Solve for R : $w = \frac{2PR}{R-r}$

27. Solve $T = \frac{1}{a} + t$ for a .

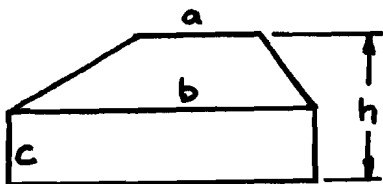
28. Solve $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$ for R .

29. Solve for E :

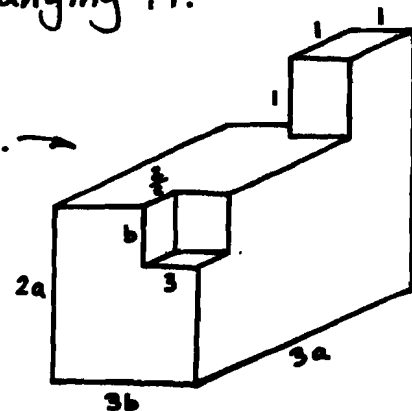
$$b + \frac{NE}{E+NR} = a - p + b$$

30. A trapezoid sits on a rectangle as shown below.

Find c in terms of a, b, h and A by writing a formula for A , the total area, and then rearranging it.



31. Write an expression for the volume V of the object shown. Now solve for b .

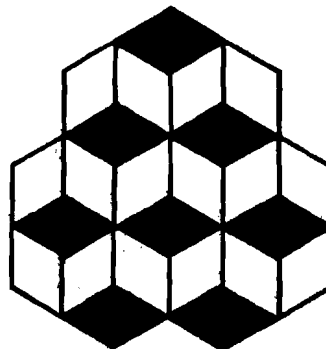


32. Solve for R : $\pi R \sqrt{\frac{1}{R^2} + H^2} = A$

Why do they call them literal expressions?

Literal means they contain letters of the alphabet, as in literature.

LO head VE heels



How many blocks do you see in this picture? Six or seven?

Note: An alternative for solving systems of linear equations is to use determinants. We are not including this in our course of study but if you are interested in investigating it, read Chapter 19 in the text.

In approximately 250 B.C. *Chiu-chang suan-shu* (9 Chapters on the Mathematical Art) was published in China. Chapter 8 included the solution of simultaneous linear equations.



In $m = \frac{k-3t}{3}$ don't cancel the 3's.

In a fraction you can only cancel factors top and bottom. If it turned out that $m = \frac{\cancel{3}(k-3t)}{\cancel{3}}$, which it doesn't, then

you could cancel the 3's as shown.

Quick Test

1. Solve the following in your head: a) $x + 2 = -3$ _____ b) $5 + x = -4$ _____ c) $2 = a + 4$ _____

d) $0 = y - 9$ _____ e) $-3 = k - 4$ _____ f) $23 - 7x = -5$ _____ g) $-4x - 2 = 10$ _____

h) $t - 8 = 3t$ _____ i) $7p - 9 = 1 - 10$ _____ j) $3(h - 2) = 6$ _____

k) $\frac{Z}{R} = -8$ _____ l) $1.23 - n = 0.06$ _____

2. Solve (for a challenge, try these in your head): a) $\frac{a}{6} + \frac{1}{2} = 1$ _____ b) $A + 4 - 0.2A = 8$ _____

c) $3Q - \frac{5}{9} = \frac{2Q}{9}$ _____ d) $\frac{4\pi(r+2)}{2} = 6\pi$ _____

3. Write an equation in 1 unknown that would lead to the solution of:

a) The average of 3 values is 38.5. If 2 of them are 40.0 and 35.0, what is the third? _____

b) Find 5 consecutive even integers whose sum is -240. _____

c) A man is 27 years older than his son. The sum of their ages is 45. How old is each? _____

d) The length of a rectangle is 5 cm less than four times its width. Find the dimensions if the perimeter is 35 cm. _____

A vase or 2 faces?

4. When manipulating equations in order to solve them, there is one important, basic rule that must be followed. What is it? _____

5. (a) When faced with an equation involving fractions, it is usually best to 1st eliminate the fractions by _____

(b) Do this for the equation below and show the resulting equation. (Do not solve.)

$\frac{Z}{8} - \frac{1}{4} = \frac{2}{5}b - 3b$ _____

Solve the following equations. (For a challenge try to do them in your head.)

6. $x + y = 2$ $x =$ _____ 7. $4x - z = -y$ $x =$ _____ 8. $2a - 4b = 7$ $a =$ _____

$x - y = -8$ $y =$ _____ $z + 2 = 1$ $y =$ _____ $6a - 12b = 10$ $b =$ _____

$x + y = 4$ $x =$ _____ $z - y = 4$ $z =$ _____

$x = y - 1$ $y =$ _____ (Use substitution)

WETHER

Write a system of equations that would allow the solution of each of the following. Do not solve.

1. A collection of 87 coins, consisting of dimes and nickels, is worth \$5.65. Find the number of coins of each type. _____

2. How much of a 15% solution of alcohol must be mixed with 2 L of 25% solution? _____

(c) If 5 green balls and 2 red balls have a mass of 3000 g, and 2 green balls and 5 red balls have a mass of 4000 g, what is the mass of each type of ball?

(d) A plane requires $5\frac{1}{2}$ h for a 2750 km trip against the wind and only 5 h for the return trip with the same wind. Find the speeds of the wind and the plane in still air.

11. Solve for the unknown indicated: a) $V_t = V_0(1 + 0.00367t)$ (for t) b) $W = RI^2t$ (for I)

c) $T = \frac{R-K}{aK}$ (for K) d) $E = 1 - \frac{T_1}{t}$ (for T_1)

Answers - Quick Test



11. (a) $t = \frac{V_0}{V_0 - V_t} - 1 = \frac{0.00367}{0.00367 - V_t}$
 (b) $I = \sqrt{\frac{W}{Rt}}$
 (c) $K = \frac{aT}{1+T}$
 (d) $T_1 = t - Et$
 (d) $\frac{2750}{5.5} = P - W$ or $2750 = 5.5(P - W)$
 (a) $n + d = 87, 5n + 10d = 565$
 (b) $x + y = 50, 0.15x + 0.40y = 0.25(50)$
 (c) $5g + 2r = 3000, 2g + 5r = 4000$
 6. $x = -3, y = 5$
 7. $x = 1, y = -5, z = -1$
 8. no solution (parallel lines)
 9. $x = 3/2, y = 5/2$
 5. (a) Multiply through by the LCD. (b) $(x+4) \rightarrow 35b - 10 = 16b - 120b$
 4. Do the same thing (+, -, x, ÷) to each side of the equation.
 (d) $w = \text{width}, \therefore \text{length} = 4w - 5 \rightarrow 2w + 2(4w - 5) = 35$
 or son's age = a, \therefore father's age = $45 - a \rightarrow (45 - a) - a = 27$
 (c) son's age = a, \therefore father's age = $a + 27 \rightarrow a + (a + 27) = 45$
 (b) m = middle value $\rightarrow (m - 4) + (m - 2) + m + (m + 2) + (m + 4) = -240$
 3. (a) n = unknown value $\rightarrow \frac{40.0 + 35.0 + n}{3} = 38.5$
 1. (a) -5 (b) -9 (c) -2 (d) 9 (e) 1 (f) 4 (g) -3 (h) -4 (i) 0 (j) 4 (k) -7/8 (l) 1.17 (m) 2. (a) 3 (b) 5 (c) 1/5 (d) 0.2 (e) 1



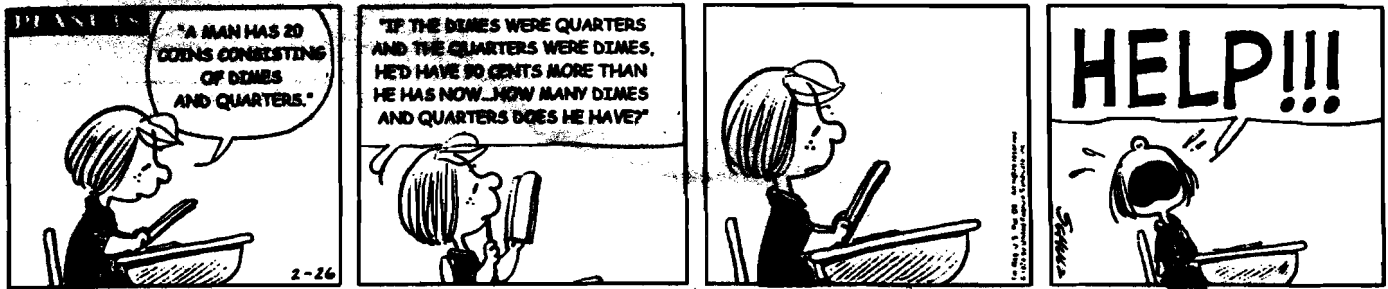
Look carefully at this dog. Can you find its master?

Readiness Test

- Solve: a) $11 - \frac{x-4}{3} = x - \frac{3x}{5}$ b) $0.011 - 0.21B = 0.0003B - 0.2B + 0.217$ c) $0.4(Q - 5) - 2(Q + 0.25) = 1.56(3 - 0.7Q)$
- Solve $(X + A^2Y^{-1})(Y - B) = CX$ for X if $A = 0.239, B = -6.66 \times 10^{-2}, C = 5.42 \times 10^{-1}$ and $Y = 3.75 \times 10^{-3}$.
- Solve using the method indicated:

(a) $5x + 4y = -7$	(b) $0.3a = 0.7b + 0.4$	(c) $1.5x + 0.3y = 0.7$
3x - 5y = -6	0.5b = 0.7 - 0.2a	y = -0.6x + 1.4
(substitution)	(addition or subtraction)	(addition or subtraction)
- a) Solve by substitution: $F_2 \sin 73^\circ - F_1 \sin 85^\circ = 0$
 (to 3 sig. figs.) $F_2 \cos 73^\circ + F_1 \cos 85^\circ - 6.00 = 0$ b) Solve: $\frac{2}{9}a + \frac{1}{7}b = 17$
 $\frac{5}{7}b - \frac{2}{3}a = -59$
- Solve using equations in 1 unknown: a) A pile of dimes and quarters has a value of \$4.60. If there are 3 more quarters than dimes, how many quarters are there?
 b) A woman's age is now 7 years more than 3 times her son's present age. In 8 years she will be 6 times as old as her son was 2 years ago (i.e. from the present). How old is each now?

6. Solve using equations in 2 or 3 unknowns: a) HELP Peppermint Patty.

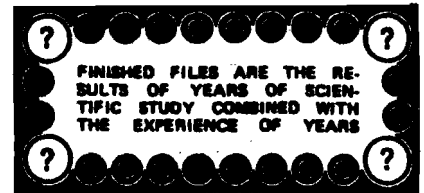


b) At a ball game a total of 1545 tickets were sold at \$1.20, \$1.80 and \$2.50 each. The number of \$1.80 tickets was 125 more than the number of \$1.20 tickets. If the total amount collected was \$2715, how many tickets of each kind were sold?

c) Mix 1 contains 65% sand and 35% cement. Mix 2 consists of 20% sand and 80% cement. How much of each must be mixed to obtain 140 kg of mix with a 45% sand content?

d) Three solutions of 4%, 28% and 60% concentrations are to be mixed to make 100 cm³ of a 24% solution. How much of each should be used if the mix is to contain twice as much of the 4% solution as of the 28% solution?

7. Zoo Story: Student: How many birds and beasts do you have?
 Zookeeper: There are 30 heads and 100 feet.
 Student: I can't tell from that.
 Zookeeper: Oh, yes you can!
 Can you?



8. Solve for the unknown indicated: a) $mx - j = hx - m$ (for m and for x) b) $wf = \frac{(w-1)}{k} \frac{1}{k}$ (for w)

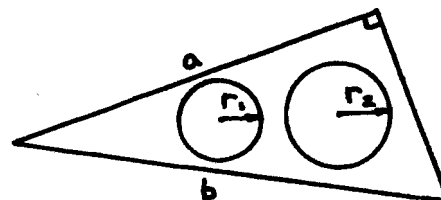
c) $k - 2 = \frac{m-2}{d-2}$ (for d) d) $\frac{2-2m}{m} + \frac{k-m^2}{m} = \frac{(m-1)^2}{m}$ (for m) e) $b^2 - \sqrt{2y-3} = m^3$ (for y)

f) $a^2 + (1-p)b^2 = a^2 + 2ab(1-p) + (1-p)^2b^2$ (for a if $p \neq 1$ and $b \neq 0$)

Using only your eyes, count the number of letter "F"s in the above sentence.

9. A triangular piece of metal has 2 holes of radii r_1 and r_2 cut out of it.

- a) Write a formula for A, the area of metal left, in terms of a, b, r_1 and r_2 .
- b) Solve this formula for b. (Don't simplify the answer.)



Questions from past tests:

10. Solve for the unknown: a) $\frac{2}{6} [k - (2-3k)^2] + \frac{k}{8} = 7 - \frac{6k^2-3}{2}$ b) $5(m-2) - m(4-m) = (m-1)^2 - 11$

c) $v^2 - \frac{v-3}{4} + \frac{2v}{6} = (v-3)^2 + \frac{v+4}{8}$ d) $a - (2-3a)^2 + \frac{a}{8} = 7 - \frac{27a^2-3}{3}$

e) $(0.3R - \frac{1}{R})(1.9R) - 3.26 = (6R-2)(0.095R-3)$ f) $(3W-2)^2 - \frac{5(W-2)}{2} - \frac{3W-2}{12} - W^2 = \frac{W}{6} + (2W-5)(4W+1)$

11. Solve for the unknowns: a) $2.4h - 3.0x = 2.4$
 Use substitution in a and b) $2.0h + 1.5x = 6.8$

b) $1.58m + 41.5 = 38.0 + 2.00n$
 $0.424m + 36.4 = 189 + 0.728n$

$5q = 75$
 $3p = 15$
 $2r = 180$

d) $a + b + c = 90$
 $2a - 3b = -20$
 $2a + 3c = 145$

e) $x - 2y + 3z = 7$
 $3x - 7y = -27$
 $x - 2y = -8$

f) $2a - b = 1$
 $c + 4a = -5$
 $2b - c = 4$

O V A T I O N

12. Solve using an equation in 1 unknown: a) The perimeter of an isosceles triangle is 50 cm. The base is 11 cm longer than each of the equal sides. Find the length of the sides.

b) In a certain fraction the denominator (bottom) is 1 less than twice the numerator. If 3 is added to the numerator and 3 is subtracted from the denominator, the fraction becomes $\frac{7}{4}$. Find the fraction.

c) How many kilograms of sand containing 7.5% water should be mixed with gravel containing 2.0% water to obtain 2500 kg of a mix containing 3.75% water?

d) The largest angle in a triangle is $2\frac{1}{2}$ times the middle angle. The smallest angle is 30° less than the difference between the other 2 angles. Determine the angles.

e) If the largest of 3 consecutive even numbers is subtracted from the sum of the other 2, the result equals half of the largest number. Find the numbers.

13. Solve using equations in 2 or 3 unknowns: a) The sum of 3 numbers is 4 times the value of the second. The third is 14 more than the sum of the other two. If $\frac{1}{2}$ of the second is added to the third, the result is 13 less than 5 times the first. Find the numbers.

b) Peewee lives 10 km from school. One morning he decides to ride his bicycle part way and run the rest. He can average 36 km/h on his bike and run at 12 km/h. If he must get to school in $\frac{1}{4}$ an hour, how far should he run and how far should he ride?

c) If the width of a rectangle is increased by 2 m and the length is increased by 12 m, the area increases by 480 m^2 . If the width is increased by 12 m and the length is increased by 2 m, the area increases by 660 m^2 . Find the original length and width.

d) Two fractions each have a denominator (bottom) of 4. They total 3. Three times the smaller fraction subtracted from five times the larger one gives 9. Find the 2 fractions.

e) On one day pipe 1 flows for $3\frac{1}{2}$ h and pipe 2 for $4\frac{1}{2}$ h and they deliver 117 000 L of oil. On another day they deliver 151 200 L of oil with the first pipe operating for 5.2 h and the second for 4.8 h. Find the flow rates in L/h for each pipe.

14. Solve for the unknown indicated. a) $mP - k^2 = \frac{-m^2h}{h-2} - 1$ (for h) b) $b - 2d = \frac{(b-d)(b-3a)}{b-a}$ (for d)

BUR

c) $\frac{1-C}{C} = \frac{A(t-T)}{2t-C}$ (for t)

Answers - Readiness Test

1. a) $(x|5) \rightarrow 165 - 5(x-4) = 15x - 9x \rightarrow x = \frac{185}{11}$ b) $\rightarrow -0.01038 = 0.206 \rightarrow B = \frac{0.206}{-0.0103} = -20$

c) $\rightarrow 0.4Q - 2 - 2Q - 0.5 = 4.68 - 1.092Q \rightarrow -0.508Q = 7.18 \rightarrow Q = \frac{7.18}{-0.508} = -14.13$

2. $\left[X + 0.239^2 \left(\frac{1}{3.75 \times 10^{-3}} \right) \right] \left[3.75 \times 10^{-3} - (-6.66 \times 10^{-2}) \right] = 5.42 \times 10^{-1} X$

$\therefore (X + 15.232)(0.07035) = 0.542X \rightarrow 0.4716X = 1.072 \rightarrow X = \frac{1.072}{0.4716} = 2.27$

3. a) $\textcircled{1} \rightarrow 5x = -4y - 7 \rightarrow x = \frac{-4y-7}{5}$

subst. in $\textcircled{2} \rightarrow 3\left(\frac{-4y-7}{5}\right) - 5y = -6$

$(x|5) \rightarrow -12y - 21 - 25y = -30$

$\therefore -37y = -9 \rightarrow y = \frac{9}{37}$

$\therefore x = \frac{-4\left(\frac{9}{37}\right) - 7}{5} = \frac{-295}{37 \times 5} = -\frac{59}{37}$

$$\begin{aligned} x &= -\frac{52}{37} \\ y &= \frac{9}{37} \end{aligned}$$

b) $\textcircled{1} \times 5 \rightarrow 1.5a = 3.5b + 2.0$

$\textcircled{2} \times 7 \rightarrow 1.4a = -3.5b + 4.9$

add $2.9a = 6.9 \rightarrow a = \frac{6.9}{2.9} = 2.379$

\therefore from $\textcircled{2} \rightarrow 0.5b = 0.7 - 0.2(2.379) = 0.2242$

$\therefore b = \frac{0.2242}{0.5} = 0.448$

$$\begin{aligned} a &= 2.38 \\ b &= 0.448 \end{aligned}$$

3. (a) ① $\rightarrow 1.5x + 0.3y = 0.7$
 ② $\times 0.3 \rightarrow 0.45x + 0.09y = 0.21$
 subtract $\rightarrow 1.05x = 0.49$
 $\therefore x = \frac{0.49}{1.05} = 0.466$
 subst. in ① $\rightarrow y = 1.27$

$x = 0.212$
 $y = 1.27$

4. (a) ① $\rightarrow 0.9563F_2 - 0.9962F_1 = 0 \rightarrow F_1 = \frac{0.9563F_2}{0.9962}$
 ② $\rightarrow 0.2924F_2 + 0.08716F_1 - 6 = 0 \therefore F_1 = 0.9599F_2$ ③
 ③ into ② $\rightarrow 0.2924F_2 + 0.08366F_2 = 6$
 $\therefore 0.3761F_2 = 6 \rightarrow F_2 = 15.95$
 $\therefore F_1 = 0.9599 \times 15.95 = 15.31$

$F_1 = 15.3$
 $F_2 = 16.0$

4. (b) ① $\times 63 \rightarrow 14a + 9b = 1071$
 ② $\times 21 \rightarrow 15b - 14a = -1239$
 add $\rightarrow 24b = -168 \rightarrow b = -7$

① $\frac{2}{3}a - 1 = 17$
 $\therefore \frac{2}{3}a = 18 \rightarrow a = 18 \times \frac{3}{2} = 27$
 $a = 81, b = -7$

5. (a) let $d = \text{no. of dimes}$
 $\therefore \text{no. of quarters} = d + 3$
 $\therefore 0.1d + 0.25(d + 3) = 4.60$
 $\therefore d = 11 \rightarrow \therefore \text{there are 14 quarters}$

(b) let $b = \text{son's age now} \rightarrow \therefore 2 \text{ years ago he was } b - 2$
 $\therefore \text{mother's age now} = 3b + 7 \rightarrow \therefore \text{in 8 years she will be } 3b + 15$
 $\therefore 3b + 15 = 6(b - 2) \rightarrow b = 9$
 $\therefore \text{son is 9, mother is 34}$

6. (a) let $d = \text{no. of dimes}$ and $q = \text{no. of quarters}$
 $\therefore d + q = 20$ ①
 $0.25d + 0.19q = 0.1d + 0.25q + 0.9$
 $\therefore \text{he has 13 dimes and 7 quarters}$

(b) let numbers of \$1.20, \$1.80 and \$2.50 tickets be a, b and c respectively
 $\therefore a + b + c = 1545$ ①
 $b = a + 125$ ②
 $1.2a + 1.8b + 2.5c = 2715$ ③
 $\therefore \text{sold 530 at } \$1.20, 655 \text{ at } \$1.80, 360 \text{ at } \2.50

(c) let mass of Mix 1 = a (kg)
 let mass of Mix 2 = b (kg)
 $\therefore a + b = 140$ ①
 $0.65a + 0.20b = 0.45 \times 140$
 $\therefore \text{use 77.8 kg of Mix 1 \& 62.2 kg of Mix 2}$

(d) let a, b, c be amounts of 4%, 28% and 60% solutions respectively (cm³)
 $\therefore a + b + c = 100$
 $a = 2b$
 $0.04a + 0.28b + 0.6c = 0.24(100)$
 $\therefore \text{use } 50 \text{ cm}^3 \text{ of 4\% solution \& } 25 \text{ cm}^3 \text{ of each of other 2}$

7. let $a = \text{no. of birds}$, $b = \text{no. of beasts}$
 $\therefore a + b = 30$
 $2a + 4b = 100$
 $\therefore 10 \text{ birds \& } 20 \text{ beasts}$

B. (b) $wf = \frac{w}{k^2} - \frac{1}{k}$
 $(\times k^2) \rightarrow wfk^2 = w - k$
 $\therefore w(fk^2 - 1) = -k$
 $\therefore w = \frac{-k}{fk^2 - 1} = \frac{k}{1 - fk^2}$

8. (a) $\rightarrow mx + m = hx + j$ and $mx - hx = j - m$
 $\therefore m(x + 1) = hx + j$
 $\therefore m = \frac{hx + j}{x + 1}$
 $\therefore x(m - h) = j - m$
 $\therefore x = \frac{j - m}{m - h}$

(c) $\rightarrow dk - 2d - 2k + 4 = m - 2$
 $\therefore d(k - 2) = m + 2k - 6$
 $\therefore d = \frac{m + 2k - 6}{k - 2}$

(d) $(xm) \rightarrow 2 - 2m + k - m^2 = m^2 - 2m + 1$
 $\therefore k + 1 = 2m^2$
 $\therefore m^2 = \frac{k + 1}{2} \rightarrow m = \sqrt{\frac{k + 1}{2}}$

THOUGHT AN

(e) $b^2 - m^3 = \sqrt{2y - 3}$
 $\therefore 2y - 3 = (b^2 - m^3)^2$
 $\therefore 2y = (b^2 - m^3)^2 + 3$
 $\therefore y = \frac{(b^2 - m^3)^2 + 3}{2}$

(f) $(1 - p)b^2 = 2ab(1 - p) + (1 - p^2)b^2$ [subtr. a^2 from both sides]
 $b = 2a + (1 - p)b$
 $\therefore b = 2a + b - pb$
 $\therefore 2a = pb \rightarrow a = \frac{pb}{2} = \frac{1}{2}pb$

(g) 3rd side = $\sqrt{b^2 - a^2} \rightarrow \therefore A = \frac{1}{2}a\sqrt{b^2 - a^2} - \pi r_1^2 - \pi r_2^2$
 $\frac{1}{2}a\sqrt{b^2 - a^2} = A + \pi r_1^2 + \pi r_2^2 \rightarrow \sqrt{b^2 - a^2} = \frac{2(A + \pi r_1^2 + \pi r_2^2)}{a}$
 both sides $\rightarrow b^2 - a^2 = \frac{4(A + \pi r_1^2 + \pi r_2^2)^2}{a^2}$
 $\frac{4(A + \pi r_1^2 + \pi r_2^2)^2}{a^2} + a^2 \rightarrow b = \sqrt{\frac{4(A + \pi r_1^2 + \pi r_2^2)^2}{a^2} + a^2}$

10. (a) $\frac{1}{3}[k - (4 - 12k + 9k^2)] + \frac{k}{8} = 7 - \frac{6k^2 - 3}{2}$
 $(\times 24) \rightarrow 8(k - 4 + 12k - 9k^2) + 3k = 168 - 72k^2 + 36$
 $\therefore 8(13k - 4 - 9k^2) + 3k = 204 - 72k^2$
 $\therefore 104k - 32 - 72k^2 + 3k = 204 - 72k^2$
 $\therefore 107k = 236 \rightarrow k = \frac{236}{107} = 2.206$

(b) $5m - 10 - 4m + m^2 = m^2 - 2m + 11 - 11$
 $\therefore 3m = 0$
 $\therefore m = 0$

(c) $\rightarrow a - (4 - 12a + 9a^2) + \frac{a}{8} = 7 - 9a^2 + 1$
 $\therefore a - 4 + 12a - 9a^2 + \frac{a}{8} = 8 - 9a^2$
 $\therefore 13a + \frac{a}{8} = 12$
 $(\times 8) \rightarrow 104a + a = 96 \rightarrow 105a = 96$
 $\therefore a = \frac{96}{105} = 0.9143$

(c) $(\times 24) \rightarrow 24v^2 - 6v + 18 + 8v = 24(v^2 - 6v + 9) + 3v$
 $\therefore 24v^2 + 2v + 18 = 24v^2 - 144v + 216 + 3v + 12$
 $\therefore 143v = 210 \rightarrow v = \frac{210}{143} = 1.469$

(e) $\rightarrow 0.57R^2 - 1.9 - 3.26 = 0.57R^2 - 18R - 0.19R + 1$
 $\therefore -11.16 = -18.19R$
 $\therefore R = \frac{-11.16}{-18.19} = 0.6135$

(f) $\rightarrow 9w^2 - 12w + 4 - \frac{5w - 10}{2} - \frac{3w - 2}{12} - w^2 = \frac{w}{6} + 8w^2 - 18w - 5$
 $(\times 12) \rightarrow -144w + 48 - 30w + 60 - 3w + 2 = 2w - 216w - 60$
 $\therefore 37w = -170 \rightarrow w = -\frac{170}{37} = -4.595$

Dice
Dice

11. (a) ② $2.0h = 6.8 - 1.5x$
 $\therefore h = 3.4 - 0.75x$ ③
 subst. in ① $\rightarrow 2.4(3.4 - 0.75x) - 3.0x = 2.4$
 $\therefore 8.16 - 1.8x - 3.0x = 2.4$
 $\therefore 5.76 = 4.8x \rightarrow x = \frac{5.76}{4.8} = 1.2$
 from ③ $\rightarrow h = 3.4 - 0.75 \times 1.2 = 2.5$
 $x = 1.2, h = 2.5$

(b) ① $\rightarrow 2.00n = 1.58m + 3.5$
 $\therefore n = 0.79m + 1.75$ ③
 subst. in ② $\rightarrow 0.424m = 152.6 + 0.728(0.79m + 1.75)$
 $\therefore -0.1511m = 153.87$
 $\therefore m = \frac{153.87}{-0.1511} = -1018$
 ③ $\rightarrow n = -803$ $m = -1018, n = -803$

(c) ③ - ② $\rightarrow 3r + 3q = 165$ ④
 ④ $\times 3 \rightarrow 9r + 9q = 495$
 ① $\rightarrow 9r - 5q = 75$
 subtr. $\rightarrow 14q = 420 \rightarrow q = 30$
 ① $9r - 150 = 75 \rightarrow r = 25$
 ② $3p - 90 = 15 \rightarrow p = 35$
 $p = 35, q = 30, r = 25$

(d) ② $\rightarrow b = \frac{2a + 20}{3}$ ④ $\rightarrow c = \frac{145 - 2a}{3}$ ⑤
 ① $\rightarrow a + \frac{2a + 20}{3} + \frac{145 - 2a}{3} = 90$
 $(\times 3) \rightarrow 3a + 2a + 20 + 145 - 2a = 270$
 $\therefore 3a = 105 \rightarrow a = 35$ $a = 35, b = 30, c = 25$
 ④ $\rightarrow b = 30$ ⑤ $c = 25$

(e) ② $\rightarrow 3x - 7y = -27$
 ③ $\times 3 \rightarrow 3x - 6y = -24$
 subtr. $\rightarrow -y = -3 \rightarrow y = 3$
 ③ $\rightarrow x - 6 = -8 \rightarrow x = -2$
 ① $\rightarrow -2 - 6 + 3z = 7 \rightarrow z = 5$
 $x = -2, y = 3, z = 5$

(f) ② + ③ $\rightarrow 4a + 2b = -1$ ④
 ① $\times 2 \rightarrow 4a - 2b = 2$ ⑤
 add $\rightarrow 8a = 1 \rightarrow a = \frac{1}{8}$
 ④ - ⑤ $\rightarrow 4b = -3 \rightarrow b = -\frac{3}{4}$
 ② $\rightarrow c + \frac{1}{8} = -5 \rightarrow c = -5 - \frac{1}{8} = -5\frac{1}{8}$
 $a = \frac{1}{8}, b = -\frac{3}{4}, c = -5\frac{1}{8}$

12. (a) let $a =$ length of equal sides (in cm) $\rightarrow \therefore$ base $= a + 11$ (cm)
 $\therefore a + a + a + 11 = 50$
 $\therefore 3a + 11 = 50 \rightarrow 3a = 39 \rightarrow a = 13 \rightarrow \therefore$ sides are 13 cm, 13 cm, 26 cm

(b) let fraction be $\frac{a}{b}$
 $\therefore b = 2a - 1$
 and $\frac{a+3}{b-3} = \frac{7}{4}$
 $\therefore 4(a+3) = 7(b-3)$
 $4a + 12 = 7(2a - 1 - 3)$

$4a + 12 = 14a - 28$
 $\therefore 10a = 40 \rightarrow a = 4$
 $\therefore b = 7$
 \therefore the fraction is $\frac{4}{7}$

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12. (c) let g = mass of gravel (kg) $\rightarrow \therefore$ mass of sand = $2500 - g$ (kg)
 $\therefore 0.075(2500 - g) + 0.02(g) = 0.0375(2500)$
 $\therefore 187.5 - 0.075g + 0.02g = 93.75 \rightarrow 93.75 = 0.055g \rightarrow g = \frac{93.75}{0.055} = 1705$
 \therefore use 795 kg of sand (and 1705 kg of gravel)

(d) let m° = middle angle $\rightarrow \therefore$ largest angle = $2.5m$ and smallest = $2.5m - m - 30$
 $\therefore 2.5m + m + 2.5m - m - 30 = 180 \rightarrow 5m = 210 \rightarrow m = 42 \rightarrow \therefore$ angles are $105^\circ, 42^\circ, 33^\circ$

(e) let numbers be $m-2, m, m+2 \rightarrow \therefore m-2 + m - (m+2) = \frac{1}{2}(m+2)$
 $\therefore m-4 = \frac{1}{2}(m+2) \rightarrow 2m-8 = m+2 \rightarrow m = 10 \rightarrow \therefore$ numbers are 8, 10, 12

13. (a) let numbers be a, b, c
 $\therefore a+b+c = 4b$ ① $\rightarrow a-3b+c = 0$ ①
 $c = a+b+14$ ② $\rightarrow a+b-c = -14$ ② \rightarrow ② $\times 2 \rightarrow 2a+2b-2c = -28$
 $\frac{1}{2}b+c = 5a-13$ ③ $\rightarrow 10a-b-2c = 26$ ③ \rightarrow ③ $\rightarrow 10a-b-2c = 26$

① + ② $\rightarrow 2a - 2b = -14$ ④
 subtr. $\rightarrow 8a - 3b = 54$
 ④ $\times 4 \rightarrow 8a - 8b = -56$
 subtr. $\rightarrow 5b = 110 \rightarrow b = 22$
 from ④ $a = 15$
 from ② $c = 51$ \therefore numbers are 15, 22, 51

PbPipe

(b) let r = distance he ran (km) and b = distance he biked (km)
 $\therefore r+b = 10$
 $\frac{r}{12} + \frac{b}{36} = 0.5 \rightarrow 3r+b = 18$ } $r = 4$
 \therefore he biked 6 km and ran 4 km

(c) let original length = L (m) and width = W (m)
 $\therefore (L+12)(W+2) = LW+480 \rightarrow 12W+2L = 456$
 $(L+2)(W+12) = LW+660 \rightarrow 2W+12L = 636$ } $W = 30$
 \therefore the rectangle was 30 m wide by 48 m long

(d) let fractions be $\frac{a}{4}$ and $\frac{b}{4}$
 $\therefore \frac{a}{4} + \frac{b}{4} = 3$ ①
 $5(\frac{a}{4}) - 3(\frac{b}{4}) = 4$ ②
 ① $\rightarrow a+b = 12$ } $a = 9$
 ② $\rightarrow 5a-3b = 16$ } $b = 3$
 \therefore fractions are $\frac{9}{4}$ and $\frac{3}{4}$

(e) let flow in pipe 1 = a (L/h) and in pipe 2 = b (L/h)
 $\therefore 3.5a + 4.5b = 117000$ } $a = 18000$
 $5.2a + 4.8b = 151200$ } $b = 12000$ \therefore 18000 L/h in pipe 1, 12000 L/h in pipe 2

14. (a) $\rightarrow (mP - k^2)(h - 2) = -m^2h - 1(h - 2)$
 $\therefore mPh - 2mP - k^2h + 2k^2 = -m^2h - h + 2$
 $\therefore mPh - k^2h + m^2h + h = 2 + 2mP - 2k^2$
 $\therefore h(mP - k^2 + m^2 + 1) = 2 + 2mP - 2k^2$
 $\therefore h = \frac{2 + 2mP - 2k^2}{mP - k^2 + m^2 + 1} = \frac{2k^2 - 2 - 2mP}{k^2 - mP - m^2 - 1}$

(b) $(b-2d)(b-a) = (b-d)(b-3a)$
 $\therefore b^2 - ab - 2bd + 2ad = b^2 - 3ab - bd + 3ad$
 $\therefore 2ab = bd + ad$
 $\therefore d(a+b) = 2ab \rightarrow d = \frac{2ab}{a+b}$

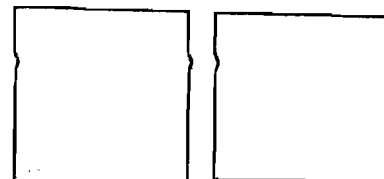


• A Japanese high-school teacher has been reprimanded after giving 100-volt electric shocks to students who forgot to bring their textbooks to class; one was left with a burn on his hand. (Electrocuting students is "absolutely wrong," says a school board official.)

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Answers - Brain Strainers

- TEN
- 6 4 9
- It's $5\frac{1}{2}$ inches long.



Which one is the square?